CMPT 250
Midterm 1

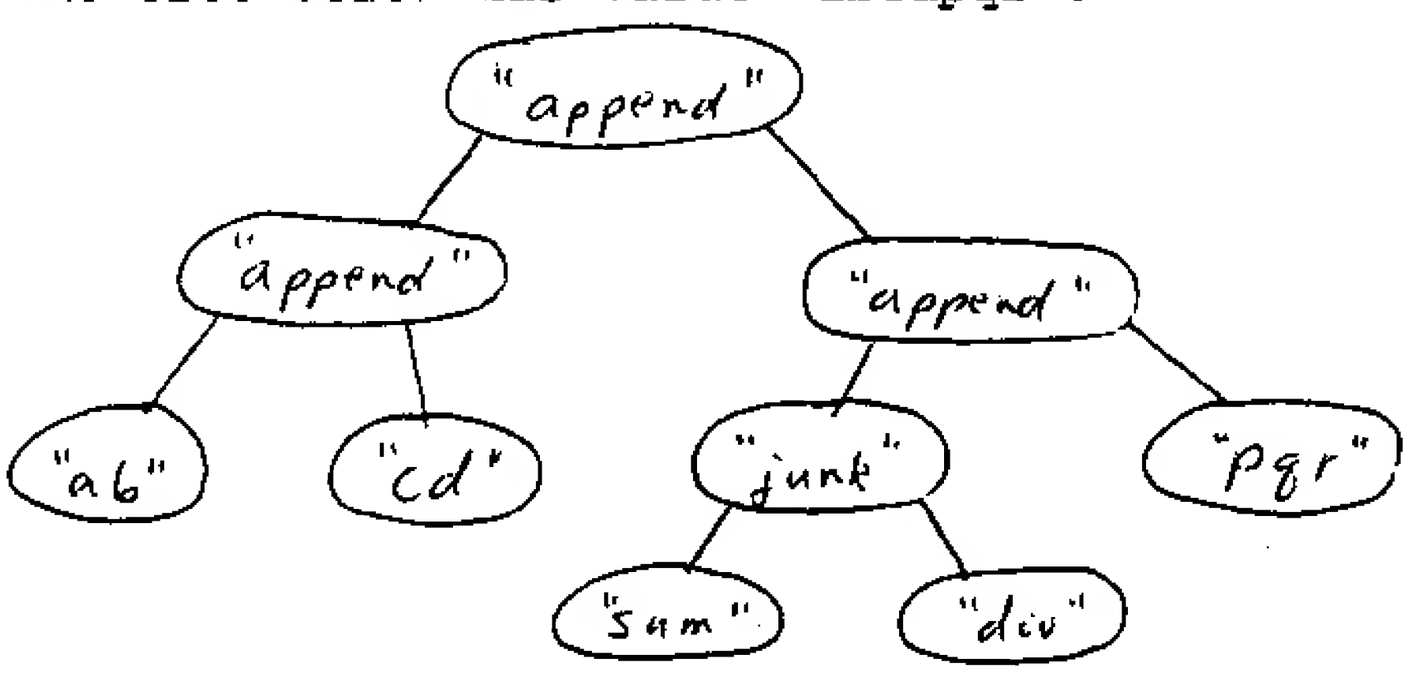
55 minutes | Closed Book | Nov. 26, 1997

Marks

3 1. Robustness is a term used to describe a piece of software. To what does it refer?

20 2. Give the Eiffel code to define a descendant of BASIC_BINARY_TREE_UOS[G], called STR_TREE, to store strings. Each tree represents a string expression. Instances of STR_TREE are to be created by the standard *make* and *set_values* routines of BASIC_BINARY_TREE_UOS[G]. In addition to the inherited routines, it is to have one new STRING function called *value*. This function is to return the STRING value of the expression represented by the tree. The value of a tree is obtained as follows. The value of a leaf node is the string stored in the node. The value of an interior node storing the string "append" is the value of its right subtree appended to the end of the value of its left subtree. (Recall Eiffel STRINGs have a procedure called *append* that does this task.) The value of any other interior node should be just the empty string (independent of its subtrees).

Thus the tree below has value "abcdpqr".



For reference, the features of BASIC_BINARY_TREE_UOS[G] are:
Note that to shorten the description, BB_TREE[G] is used instead of BASIC_BINARY_TREE_UOS[G].

```
make
set_values (lt:BB_TREE[G]; x:G; rt:BB_TREE[G])
root_left_subtree : BB_TREE[G]
set_root_left_subtree (newt:BB_TREE[G])
root_right_subtree : BB_TREE[G]
set_root_right_subtree (newt:BB_TREE[G])
root_item : G
set_root_item (x:G)
insert_root (x:G)
insert_root_left (x:G)
insert_root_right (x:G)
delete_root
make_cursor : TREE_CURSOR_UOS [G]
```

8 3. For <u>each</u> of the following routines, give its time requirements using O(.) or Θ(.) notation (or both), which ever is appropriate. Express your final answer in as simple a form as possible, without losing any accuracy.

Assume that

s is a procedure with its time given by $T_s(n) \in O(\log_2 n)$.
t is an INTEGER function with time requirements given by
$T_t(m) \in \Theta(m)$.
a is an array with dimension 1 to m
b is an array with dimension 1 to n

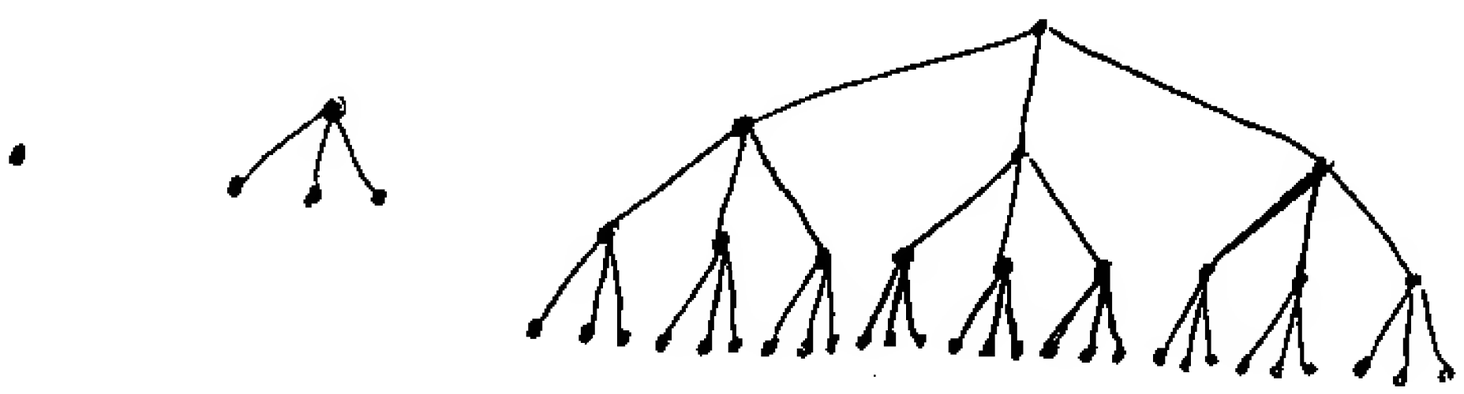
Note that these routines are not meant to do anything useful or interesting. Also, read the routines carefully so as not to miss any routine calls.

```
d (x:INTEGER) : INTEGER is
   local i:INTEGER
   do
      from i:=1
      until i>m or else a.item(i) = x
      loop

         s

         i := i + 1
      end
      Result := i
   end

p is
   local j:INTEGER
   do
      from j := 1
      until j>n
      loop
         a.put( d(j) + t(j), j)
         j := j + 1
      end

      q(n)

   end

q (k:INTEGER) is
   do
      b.put( t(2*k+1), k)
      if k-1 > 0
      then
         q(k-1)
      end
   end
```

7 4. The question is to give the axiomatic definition for an operation on ordered binary trees. You should assume that *make* and *set_values* are the build operations with the usual parameters and semantics. The operation to be done is to be called *duplicate_root*. The semantics of the operation are to insert another copy of the root item into the tree. The duplicated copy can be inserted anywhere convenient as long as the tree is still ordered. Thus if initially a tree contained the value 12 three times, one of which was at the root, then after the operation *duplicate_root*, the tree should contain the value 12 four times. The number of occurrences all other items should not change.

Give the signature, any pre-conditions, and the axioms.

12 5. Prove by induction that the number of nodes in a complete ternary tree of height h is $(3^{h+1}-1)/2$. Recall that the height of a tree is -1 for the empty tree, and otherwise the length of the longest path from the root to a leaf. Each node in a ternary tree has between 0 and 3 children. In a complete ternary tree each node has either 0 or 3 children (never 1 or 2). Also in a complete ternary tree, all leaves are the same distance from the root. Finally the empty tree is a complete ternary tree. The diagram below gives several complete ternary trees.



Total 50

The end